Quantifying Technical Efficiency of Paratransit Systems by Data Envelopment Analysis Method

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This research evaluates efficiency levels of individual paratransit systems in Canada with the specific objective of identifying the most efficient agencies and the sources of their efficiency. Through identification of the most efficient systems along with the influencing factors, new service policies and management and operational strategies might be developed for improved resource utilization and quality of services. The research applies the data envelopment analysis methodology, which is a mathematical programming technique for determining the efficiency of individual systems as compared with their peers in multiple performance measures. Annual operating data from 2001 to 2003 as reported by the Canadian Urban Transit Association are used in this analysis. A bootstrap regression analysis is performed to identify the possible relationship between the efficiency of a paratransit system and measurable operating or managerial factors that affect the performance of paratransit systems. The regression analysis allows for the calculation of confidence intervals and bias for the efficiency scores.

Demand-responsive paratransit is considered to be an important public transportation mode in most municipalities in North America, providing services to people with special requirements, such as seniors and people with disabilities. In the past 10 years, paratransit has expanded significantly in the United States and Canada in order to accommodate increased demand. Because of its door-to-door service approach with a fare scheme comparable with regular transit, most paratransit systems in North America rely heavily on subsidies to cover their operating costs. According to the American Public Transit Association, the total operating expense of paratransit services in the United States exceeded $1.2 billion with only $173 million collected in fares. The Canadian Urban Transit Association (CUTA) reported that the total operating expenses of 50 Canadian paratransit agencies amounted to approximately $150 million (Canadian dollars), of which only 10% was recovered from fare revenues. It is projected that the demand for paratransit services will increase significantly in the near future due to the aging population. This growth in demand will place even greater pressure on paratransit agencies to find ways to reduce agencies' operating costs and improve service efficiency.

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The first step toward performance improvement for the paratransit industry is to benchmark existing service levels. Specifically, the following questions may be asked. Who are the best performers in the paratransit industry? What is the maximum achievable efficiency given the demand profile and operating environment? What can be accomplished either through changing service policies or more aggressively pursuing ways to reduce vehicle hours?

To address all these questions requires a methodology to quantify the efficiency of a paratransit system and understand how it is affected by various system factors. Quantifying the performance of paratransit systems is, however, a challenging task because it is influenced by a large number of factors, many of which are outside of the control of the supplying agency. Variables—such as the size of the service area, passenger demand density, spatial and temporal distributions, and average trip length—are external inputs to the agency's decision-making process. However, many so-called managerial factors are within the control of the service providers, such as service policies (e.g., pickup windows, curb-to-curb versus door-to-door service), fleet mix (e.g., fleet size and vehicle type), trip scheduling method (i.e., manual versus computerized), and driver and dispatcher training.

This research introduces a technique called data envelopment analysis (DEA) for evaluating the efficiency of paratransit service systems as a function of both external and internal variables. In particular, this research assesses the suitability of the DEA approach for evaluating the efficiency of paratransit systems using data from the Canadian paratransit sector. This approach identifies the best performers of the Canadian paratransit systems and, if possible, the factors that are associated with these service systems.

MEASURING PARATRANSIT EFFICIENCY

As for any service systems, the planning, management, and operation of paratransit services are done to achieve a balance between the efficiency (or productivity) of the service and the quality of service experienced by passengers (*1*). By definition, the term "efficiency" reflects the quantity of output generated as a function of the inputs to the system. In paratransit, the output can be the number of passengers serviced and/or total passenger kilometers covered as a function of, for example, the number of vehicle hours, number of employees, total operating costs, or other factors. In contrast, quality of service is a measure representing the degree to which the outputs produced by the system meet the requirements of the users. In the case of paratransit, quality of service can be estimated by specific measures of the length of pickup and drop-off time windows, ride directness, or riding comfort.

Oftentimes, efficiency and service quality are competing objectives. Consider the example of exclusive-ride systems versus shared-ride systems. In the first case, a single rider is transported directly from origin to destination, providing the fastest, most convenient service. This approach, of course, is also most expensive on a dollar per passenger-kilometer basis. With shared-ride service, vehicles are routed with intermediate pickups and drop-offs such that vehicle trips are minimized. Obviously, shared-ride systems increase operating efficiency by reducing the total vehicle miles traveled and the number of vehicles required, but decrease passenger quality of service by increasing average ride time and the variability of promised pickup and arrival times. Given that demand for the service is a function of the quality of the service, paratransit operators need to be cognizant that overemphasizing efficiency at the expense of quality of service may result in greatly decreased ridership. This research focuses only on issues related to the efficiency of a paratransit system.

Efficiency of a system can be viewed from two different perspectives: economically and technically. Economic efficiency measures the relationship between the value of the output and the value of the input, and therefore can help examine the profitability of a system for an investment. When examining the economic efficiency, the value of output over the value of input provides an absolute measure of efficiency. In contrast, technical efficiency directly compares the output with the input. When a system is called inefficient, it means that one could achieve the desired output with less input, or that the input employed could produce more of the output desired. This research focuses particularly on the operational productivity of paratransit systems, that is, technical efficiency. Measures of efficiency that are commonly used in practice are listed in Table 1.

There are two main approaches that can be used to measure the technical efficiency of a system: parametric and nonparametric frontier approaches. A parametric approach specifies a functional form for relationship between system output and inputs and environmental factors. The functional relationship can be established through a regression technique using operational data obtained from the systems under consideration. Based on the functional relationship, the efficiency level that can be expected from a given system can be estimated on the basis of its input and environmental factors. This estimated efficiency can then be compared with what is actually observed, providing an indication of how efficient a system is currently operating. Past research has indicated that parametric approaches are most applicable to industries with well-defined technologies (*2*).

In contrast, a nonparametric approach does not require explicit specification of the form of the underlying production relationship and is most suitable for noncompetitive industry with imprecise technologies, such as the service sector (*3*). This following section introduces a nonparametric approach—DEA—which has been applied in this research to evaluate the technical efficiency of paratransit transit systems. A detailed discussion on DEA is provided.

TABLE 1 Common Efficiency Measures for Urban Transit

Efficiency Measure	Efficiency Indicator
Cost efficiency	Cost per km/mi Cost per hour Cost per vehicle Cost per passenger trip
Cost-effectiveness	Revenue per passenger trip Ridership per expense Passenger trips per km/mi
Service utilization efficiency	Passenger trips per hour Passenger trips per capital Km/mi per vehicle
Vehicle utilization efficiency	Passenger trips per employee
Labor productivity	Vehicle miles per employee Vehicle km/mi per capital
Coverage	Total vehicle km/mi Service areas

DEA APPROACH

DEA is commonly used to evaluate the efficiency of a group of producers (also called decision-making units or DMUs), such as organizations, firms, departments, or operating units. DEA is a nonparametric frontier approach, which was originally proposed by Farrell (*4*). The original motivation was the need for a simple measure of efficiency that is capable of accounting for multiple inputs. A typical statistical approach evaluates producers relative to an average producer. In contrast, DEA compares each producer with only the "best" producers. A fundamental assumption behind DEA is that if a given producer, *A,* is capable of producing $Y(A)$ units of output with $X(A)$ inputs, then other producers should also be able to do the same if they were to operate efficiently. Similarly, if producer *B* is capable of producing $Y(B)$ units of output with $X(B)$ inputs, then other producers should also be capable of the same production schedule. Producers *A, B,* and others can then be combined to form a composite producer with composite inputs and composite outputs. Because this composite producer does not necessarily exist, it is sometimes called a virtual producer.

DEA has gained increasing popularity as a vital tool for evaluating efficiency of almost all transportation modes. For example, Chu et al. (*5*), Karlaftis (*6*), and Boame (*7*) applied DEA to examine the technical efficiency of U.S. and Canadian public transportation systems. Oum et al. (*8*) provided an overview of various DEA applications in railroads.

The heart of the DEA technique lies in finding the "best" virtual producer for each real producer. If the virtual producer is better than the original producer by either making more output with the same input or making the same output with less input, then the original producer is inefficient. The procedure of finding the best virtual producer can be formulated as a linear program. Assume there are data on k inputs (denoted by x_i, $i = 1,2, \ldots, k$) and m outputs (denoted by y_j, $j = 1,2, \ldots, m$) on each of N producers or DMUs. The $k*n$ input matrix, X, and the $m*n$ output matrix, Y, represent the data of all N DMUs. The purpose of DEA is to construct a frontier that envelops all data points representing the efficiency of all DMUs under consideration and calculate the efficiency score for each DMU. For the simple example of an industry in which one output is produced using two inputs, it can be visualized as a number of intersecting planes forming a tight cover over a scatter of points in two-dimensional space. The problem of determining the efficiency score for each DMU_r ($r = 1,2, \ldots, N$) can be formulated as the following linear programming problem (*3*):

$$\max \theta_r = \frac{\sum_{j=1}^{m} u_j y_{r,j}}{\sum_{i=1}^{k} v_i x_{r,i}} \tag{1}$$

subject to

$$\theta_r = \frac{\sum_{j=1}^{m} u_j y_{r,j}}{\sum_{i=1}^{k} v_i x_{r,j}} \leq 1 \qquad \text{for } r = 1, 2, \ldots, N$$

where

θ_r = efficiency score for DMU_r,
$y_{r,j}$ = amount of output j produced by DMU_r,
$x_{r,i}$ = amount of input i used by DMU_r,
u_j = decision variable representing the weight for output j, and
v_i = decision variable representing the weight for input i.

Given the inputs and outputs, the linear programming problem formulated in Equation 1 can be solved for the weights (u_j and v_i) N times, once for each DMU in the sample. A value of θ for each DMU can then be calculated from the inputs, outputs, and the corresponding weights.

Because of its nonparametric feature and its ability to combine multiple inputs and outputs, DEA has been found to be a powerful tool when used appropriately. A few of the characteristics that make DEA powerful are listed below:

- DEA can handle multiple input and multiple output models.
- DEA does not require an assumption of a functional form relating inputs to outputs.
- DMUs are directly compared against a peer or combination of peers.
- Inputs and outputs can have different units.

The same features that make DEA a powerful tool can also create problems. The following limitations must be considered when choosing whether to use DEA:

- As an extreme point technique, DEA is sensitive to noise (even symmetrical noise with zero mean) such as measurement errors.
- DEA is a technique for estimating "relative" efficiency of a DMU as compared with its peers instead of a "theoretical maximum." It does not provide any information on the maximum efficiency that a DMU can achieve.
- As a nonparametric technique, DEA does not allow conventional statistical hypothesis tests.
- The standard DEA approach has the disadvantage that it cannot distinguish between changes in relative efficiency brought about by movements toward or away from the efficiency frontier in a given year and shifts in this frontier over time.
- The DEA method assigns mathematically optimal weights to all inputs and outputs being considered. It empirically derives the weights so the maximum weight is placed on those favorable variables and the minimum weight is placed on the unfavorable variables. The underlying assumption of this method is that it is equally acceptable to specialize in producing any output or consuming any input. In many cases, this type of free specialization without weight restrictions is not acceptable or desirable and may lead to unreliable conclusions.

ASSESSING CANADIAN PARATRANSIT SYSTEMS BY USING DEA

Description of Data

The data used in this research are provided by CUTA as part of the annual publication entitled *Summary of Canadian Transit Statistics Operating Data.* The published data include annual operating statistics and trends of all Canadian urban transit systems. The data used in this analysis are paratransit-specific and cover the period from January 1, 2001 to December 31, 2003.

The data in the annual publication include both publicly and privately operated transit systems providing conventional and specialized transit services to urban municipalities in Canada. Because transit services in Canada are not subsidized by the federal government, there is no uniform information requirement, and all data are submitted to CUTA on a voluntary basis. As a result, the data element definitions and accounting procedures employed by individual systems may vary considerably. Also, fare structure, service policies, subsidy levels, and the local operating environment may vary from system to system and from province to province. Therefore, caution must be taken in comparing the performance of different transit systems.

The data contain general information for both conventional transit services and specialized transit services. The data for specialized transit services are used to analyze the efficiency of paratransit systems. It should be noted that the database does not include information on community bus services and some private nonprofit paratransit services (for example, transportation service provided by Canadian Cancer Society). Table 2 provides a summary of operating statistics of all paratransit systems in Canada for the analysis period.

TABLE 2 Summary Statistics of Canadian Paratransit, 2000–2003

	2000	2001	2002	2003
Number of transit systems reporting	58	60	60	60
Total vehicle kilometers	52,465,836	52,524,934	55,649,453	55,753,517
Total vehicle hours	2,704,238	2,804,652	2,894,866	2,917,468
Passenger boardings	10,870,147	11,126,423	11,640,015	11,792,766
Total direct operating expenses	185,447,066	197,224,952	215,068,952	231,337,741
Total operating revenue	17,595,185	18,631,352	19,740,612	20,449,070
Nonaccessible cars	193	214	173	360
Accessible vans and minivans	716	677	627	794
Small buses	575	713	827	726
Total employees	2,350	2,361	2,388	2,472

Inputs and Outputs

The first step in a DEA is to identify the inputs and outputs that can be potentially used to define the efficiency of a paratransit system. Unlike many other industries in which output (e.g., consumer products) consists of clearly identifiable objects, the output of a paratransit agency (or transit agency in general) is service that may or may not be actually used by the consumers—paratransit users. For example, the seating capacity of a paratransit vehicle assigned to a route may not be fully utilized (i.e., not all seats are used), which means the service produced is not the same as the service that is actually utilized. As a result, the amount of paratransit service is often measured differently, either by the amount of service produced or by the amount of service actually "consumed" or used. An example for the amount of service produced is total vehicle kilometers serviced. Example measures for the amount of service used are total passenger kilometers and total number of passengers serviced. Because this research is concerned with technical efficiency (i.e., output produced versus input), measures for the amount of service produced are considered as system output. More specifically, a single output measure is used—revenue vehicle kilometers—which is defined as the total kilometers traveled while in revenue-generating service. The use of revenue vehicle kilometers implicitly avoids these vehicle mileages that are nonproductive and should not be counted as being contributive to system efficiency (e.g., deadheading, training, road tests, maintenance, or any auxiliary passenger services).

For system input, three input quantities (labor, fuel, and capital) are considered:

- Labor is measured as the total equivalent number of full-time employees who are hired to provide the paratransit service, including operators, maintenance, and administrative personnel. A part-time employee could be counted as one-half of a full-time employee.
- Fuel is usually measured by the total annual amount of fuel used by the system (in liters). However, the CUTA database includes only the annual fuel expenses. Therefore, this study uses fuel expenses as a measure of fuel consumption.
- Capital is the total number of vehicles used by the system.

Efficiency Model

With the inputs and outputs identified in the previous sections, the basic DEA model for a given paratransit system can be formulated as follows:

$$\max \theta_r = \frac{u_1 K_r}{v_1 V_r + v_2 F_r + v_3 E_r} \qquad (2)$$

subject to

$$\frac{u_1 K_r}{v_1 V_r + v_2 F_r + v_3 E_r} \leq 1 \text{ for all paratransit systems}$$

$$u_1 > 0; v_1, \ldots, v_3 > 0; r = 1, 2, \ldots, 29$$

where

θ_r = efficiency score of the paratransit system r,
K_r = total revenue vehicle kilometers provided by paratransit system r,
V_r = total number of vehicles used in service by paratransit system r,
F_r = total fuel expenses incurred by paratransit system r,
E_r = total number of employees hired by paratransit system r, and
u_1, v_1, v_2, v_3 = decision variables representing weighting factors for the output and input factors.

The software package GAMS (General Algebraic Modeling System, GAMS Development Corporation) was used to solve the formulated linear programming problems. Table 3 provides the solution results indicating the efficiency of individual paratransit systems in Canada for each year (note that the names of the municipalities were removed for confidentiality). From these results, the following observations can be made:

- The technical efficiency of Canadian paratransit systems varies significantly across systems with values ranging from 0.164 to

TABLE 3 Efficiency Scores of Canadian Paratransit Systems, 2001–2003

Municipality Code	2001	2002	2003	% Change, 2002–2001	% Change, 2002–2003
C1	0.601	0.635	0.654	5.66	2.99
C2	0.532	0.470	0.349	−11.65	−25.74
C3	0.855	0.633	0.655	−25.96	3.48
C4	0.883	1.000	0.984	13.25	−1.60
C5	0.841	1.000	0.751	18.91	−24.90
C6	0.164	0.168	0.232	2.44	38.10
C7	0.530	0.524	0.562	−1.13	7.25
C8	0.671	0.631	0.552	−5.96	−12.52
C9	0.798	0.736	0.772	−7.77	4.89
C10	0.777	0.760	0.754	−2.19	−0.79
C11	0.411	0.541	0.524	31.63	−3.14
C12	0.658	0.777	0.754	18.09	−2.96
C13	0.619	0.758	0.613	22.46	−19.13
C14	0.643	0.843	0.711	31.10	−15.66
C15	0.666	0.824	1.000	23.72	21.36
C16	0.598	0.680	0.609	13.71	−10.44
C17	0.684	0.716	0.665	4.68	−7.12
C18	1.000	0.504	0.481	−49.60	−4.56
C19	0.464	0.497	0.721	7.11	45.07
C20	1.000	1.000	1.000	0.00	0.00
C21	1.000	0.729	0.629	−27.10	−13.72
C22	0.644	0.782	0.895	21.43	14.45
C23	0.539	0.645	0.571	19.67	−11.47
C24	0.617	0.730	0.744	18.31	1.92
C25	0.790	1.000	0.849	26.58	−15.10
C26	0.660	0.708	0.745	7.27	5.23
C27	0.767	0.916	0.905	19.43	−1.20
C28	0.664	0.799	0.718	20.33	−10.14
C29	0.861	0.757	0.719	−12.08	−5.02
C30	0.349	0.458	0.425	31.23	−7.21
C31	1.000	1.000	1.000	0.00	0.00
C32	0.704	1.000	0.349	42.05	−65.10
Average	0.6872	0.7257	0.6841	0.0799	−0.0352
Std. dev.	0.1939	0.1955	0.1941	0.1958	0.1892
Minimum	0.164	0.168	0.232	−0.496	−0.651
Maximum	1.000	1.000	1.000	0.420	0.451

1.000. The average efficiency of all systems is 0.687, 0.725, and 0.684 for year 2001, 2002, and 2003, respectively. The variation over the 3 years is quite consistent with a standard deviation of approximately 0.29.

- Among all systems, the paratransit systems operated by agency C20 and C31 consistently outperformed other systems (100% efficient) over the 3-year period. These systems are the best performers that other paratransit systems may consider as a benchmark for improving their efficiency. This is because the efficiency score is a measure of "relative" efficiency on how well or badly a paratransit system is operated as compared with the most efficient ones.
- In terms of change in efficiency score over the 3 years, there were two systems, agency C15 and C22, that experienced noticeable increases in efficiency (over 20% increase per year).
- There were also three systems (C2, C18, and C21) whose whole efficiency scores decreased significantly (approximately −15% per year). It would be valuable to find out, for example, through a survey, what actions had been taken by these systems over these years that had led to the dramatic changes in their technical efficiency.

The paratransit service offered by system C6 is least efficient, with an efficiency score of between 0.164 and 0.232. The service by system C30 is ranked the second worst, with an efficiency score of between 0.349 and 0.458. Again, it would be interesting to examine the particular environments and service management methods associated with these two cities.

EXTERNAL FACTORS INFLUENCING EFFICIENCY

As shown in the previous section, there is a significant variation in technical efficiency across systems. To identify the sources of efficiency or inefficiency associated with these systems, it is necessary to conduct an analysis of the relationships between the efficiency scores of individual systems and their characteristics. The system characteristics are external variables that describe factors that may influence the efficiency of a paratransit system but are nontraditional inputs (or outputs). The objective of this section is to assess the impacts of various factors on the efficiency of individual paratransit systems. In particular, the following three hypotheses are presented:

- Is the level of automation in scheduling a factor influencing the efficiency of a paratransit system?
- Is the average vehicle travel speed a factor influencing the efficiency of a paratransit system?
- Is demand (e.g., density) a factor influencing the efficiency of a paratransit system?

There are several ways in which environmental variables can be linked to efficiency score in a DEA study. This paper uses the two-stage method proposed by Coelli et al. (*9*), which involves solving a DEA problem in the first-stage analysis with only the conventional inputs and outputs. In the second stage, the efficiency scores from the first stage are regressed on the environmental variables. The sign of the coefficients of the environmental variables indicates the direction of the influence, and standard hypotheses tests can be used to assess the strength of the relationship.

Advantages of this two-stage method include the ability to (*a*) accommodate more than one variable, (*b*) include both categorical and continuous variables, (*c*) remove the need to make prior assumptions regarding the direction of the influence of the variables, and (*d*) calibrate the model easily. One disadvantage of the two-stage method is that if the variables used in the first stage are highly correlated with the second-stage variables, then the result is likely to be biased.

The relationship between the efficiency level and various influencing factors can be established using a regression method based on the availability of data. The following three environmental variables are identified as the possible factors influencing paratransit efficiency:

- Schedule is a variable used to represent the level of automation in generating schedules and runs by a paratransit service provider. It is expected that higher levels of automation may generate more efficient routes and thus lead to higher technical efficiency. In this study, a manual scheduling process is rated 4, a partially computerized scheduling method is rated 2, and a fully automated scheduling process is rated 1.
- Average speed is defined as the average ratio of travel distance to travel time for all trips in the service area covered by a paratransit system. Increasing average speed allows more trips to be covered within a given time period and thus lowers outputs such as travel cost per kilometer and vehicle operating cost. An urban transit system with higher average speeds often implies fewer stops which may also reduce maintenance requirements as well. Thus higher average speeds should correlate with higher levels of technical efficiency, and vice versa.
- User area density represents the number of users per unit area. It is easy to understand that the closer the users' activities are, the more concentrated the pickup and delivery stops and the shorter the trip length. Thus the density of users in an area should be considered when efficiency is examined. This factor is calculated from service area and total number of users included in the original database.

To test the hypothesis on the significance of each of these factors, the following linear regression model is evaluated:

$$\theta_r = \beta_0 + \beta_1 \times \text{schedule}_r + \beta_2 \times \text{density}_r + \beta_3 \times \text{speed}_r + \epsilon_r \qquad (3)$$

where

θ_r = efficiency score of paratransit system r;
β_i = model coefficients to be estimated; and
ϵ_r = a normally distributed error term.

The efficiency scores obtained from the DEA process can be regressed directly against the three variables described previously using the least-squares method. However, the efficiency scores obtained from DEA may be related to each other because the efficiency of one DMU was obtained using the inputs and outputs of all other DMUs. The possible dependency among the responses (i.e., efficiencies) violates the independency assumption required by the ordinary linear regression method. An implication of this violation is that the estimated standard error of each model coefficient may not be valid, which means it cannot be used in a normal hypothesis test for testing the significance of an explanatory variable. A solution to this problem is to apply the bootstrap method to obtain multiple DEA estimates, which were then used in the subsequent regression analysis to obtain model coefficients. This bootstrapping approach has been successfully applied to obtain valid standard errors for the regression coefficients (*10*). The bootstrap regression method, as applied in this research, has the following steps:

1. Construct a sample probability distribution for each DMU of the observed 32 paratransit systems.

2. Generate 2,000 random samples of size 32 with replacement from the observed sample of 32 paratransit systems. These samples are the bootstrap samples.

3. Run the DEA for each bootstrap sample to obtain efficiency scores.

4. Within each bootstrap sample, fit the following linear regression model:

$$\theta_{ki} = b_{k0} + b_{k1} \times \text{schedule}_{ki} + b_{k2} \times \text{density}_{ki} + b_{k3} \times \text{speed}_{ki}$$
$$\text{for } i = 1,2,\ldots,32;\ k = 1,2,\ldots,2000 \quad (4)$$

where θ_{ki} is the DEA efficiency score for paratransit system i in bootstrap sample k, and b_{kj} is the estimated model coefficients for bootstrap sample k.

5. Estimate the standard error for b_{kj}, $s(b_{kj})$, by the sample standard deviation of all 2,000 bootstrap replications:

$$s(b_j) = \left\{ \frac{\sum_{k=1}^{2,000} \left(b_{kj} - \bar{b}_j\right)^2}{2,000 - 1} \right\}^{\frac{1}{2}} \quad (5)$$

where

$$\bar{b}_j = \frac{\sum_{k=1}^{2,000} b_{kj}}{2,000}, j = 1,\ldots,3$$

The last step is to calculate the t-statistic [$t = b_j/s(b_j)$] and then compare the calculated t to the critical value $t_{0.025}$ from the Student t-distribution with degree of freedom equal to $100 - 4 - 1 = 95$. If $|t| > t_{0.025}$, reject the null hypothesis $H_0: b_j = 0$, in favor of $H_0: b_j \neq 0$, and conclude that the jth factor influences the efficiency of paratransit system at $\alpha = 0.05$ significant level. Otherwise, the null hypothesis $H_0: b_j = 0$ is tenable and the null hypothesis that the jth factor does not influence the efficiency of the paratransit systems at $\alpha = 0.05$ significant level cannot be rejected. The bootstrap procedure was again coded in GAMS. Table 3 shows the results of calibration.

Results from both ordinary regression and bootstrap regression, as shown in Table 4, indicate that both user area density and the average speed had a statistically significant impact on the technical efficiency of a paratransit system. The positive coefficients associated with these two variables make intuitive sense because they suggest positive correlation between efficiency and demand density and average speed. The variable schedule was found to be statistically insignificant. Initially, it was thought that this result might be caused by the assignment of numerical values to the different level of automation, which did not reflect the difference in efficiency level induced by using different scheduling tools in reality. Subsequently, the authors tried to model the level of automation as a categorical variable. The result was, however, the same—that is, there was no significant difference in efficiency between systems with different scheduling methods. Although this finding is somehow consistent with some empirical results of several past studies, it does not necessarily conclude that level of automation in scheduling has no contribution to system's efficiency at all. A closer examination of the change in system efficiency as related to change in scheduling method has resulted in mixed results.

As indicated in Table 5, 10 paratransit systems changed their scheduling tool over the study period (2001–2003). Among them, the cities of C15, C24, and C30 changed their scheduling method from partially computerized to fully computerized in 2002, resulting in higher efficiency scores. The system C7 changed form fully computerized to partially computerized and the efficiency score decreased slightly. For the city of C20, its technical efficiency remained at 100% although its scheduling tool was improved. However, the remaining five systems introduced new scheduling tools but experienced lower levels of efficiency. It is unknown whether the change in scheduling tool is related to the efficiency from analysis of these changes. Further study is therefore required to confirm the effectiveness of scheduling method on the efficiency of a paratransit system.

After the insignificant factor schedule is removed, the recalibrated efficiency model takes the form:

$$\theta_r = 0.230 + 0.004 \times \text{density}_r + 0.020 \times \text{speed}_r \quad (6)$$

The regression analysis results for an efficiency model without the variable schedule are listed in Table 6.

As a final analysis, this paper examines the possible relationship between the technical efficiency of a paratransit system and its economical efficiency as represented by its revenue to cost ratio. The hypothesis was that those agencies that are more efficient technically should also be more efficient economically, with higher revenue-to-operating-cost ratios. Figure 1 presents the efficiency scores and the revenue-to-cost ratio from 2001 for all 32 agencies

TABLE 4 Results of Linear Regression Analysis, with All Three Independent Variables

Method	Estimates	SE	t-Value	p-Value
OLS regression				
Schedule	−0.003	0.021	−0.122	0.903
Density	0.004	0.001	2.973	0.004
Speed	0.020	0.003	7.264	1.19E-10
Constant	0.233	0.067	3.476	0.0078
Bootstrap method				
Schedule	−0.003	0.028	−0.091	0.927
Density	0.004	0.002	2.254	0.027
Speed	0.020	0.004	5.515	3.17E-7
Constant	0.233	0.086	2.715	0.008

TABLE 5 Scheduling Tool Changes Versus Efficiency Changes

	Efficiency			Schedule		
Community	2001	2002	2003	2001	2002	2003
C1	0.600	0.593		2	1	
C7	0.526	0.504		1	2	
C8	0.618	0.606		2	1	
C13		0.655	0.544		2	1
C15	0.664	0.730		2	1	
C17	0.680	0.666		4	1	
C20	1.000	1.000		2	1	
C21	1.000	0.654		4	2	
C24	0.612	0.703		2	1	
C30	0.349	0.407		2	1	

TABLE 6 Results of Linear Regression Analysis, with Only Significant Variables

Method	Estimates	SE	*t*-Value	*p*-Value
OLS regression				
Density	0.004	0.001	3.036	0.003
Speed	0.020	0.003	7.264	8.430E-11
Constant	0.230	0.060	3.834	2.292E-4
Bootstrap method				
Density	0.004	0.002	2.301	0.024
Speed	0.020	0.004	5.591	2.244E-7
Constant	0.230	0.076	3.036	0.003

considered in this study. Figure 2 presents the data in a slightly different way, in which efficiency scores for all 3 years are plotted on the horizontal axis versus revenue-to-cost ratio on the vertical axis. Based on these two plots, there appears to be very little correlation between technical operating efficiency and revenue-to-cost ratio, underscoring the difference between these two efficiency perspectives and the need to consider both in benchmarking different paratransit systems.

CONCLUSIONS

The paratransit agencies need tools and guidelines that can be used to benchmark their performance against their peers and to identify the important controllable factors that affect the efficiency of their service systems. This research has introduced the data envelopment analysis approach for addressing the problem of determining the technical efficiency of paratransit systems. Three years of operating data from 32 Canadian paratransit agencies were used in this analysis. The following is a list of the major findings and conclusions:

- DEA was found to be effective and relatively easy to use for quantifying the technical efficiency of paratransit systems. Based on the case study of Canadian paratransit systems, it was found that efficiency score was quite sensitive to systems with a wide spread of variation. Large variation in efficiency estimates facilitates the investigation of factors contributing to the efficiency of individual systems.
- To identify the factors that influence the technical efficiency of paratransit systems in Canada, a linear regression analysis was conducted to relate technical efficiency of paratransit systems to various environmental factors. It was found that demand density and average travel speed had a significant impact on paratransit efficiency. The regression analysis indicates that higher efficiency is associated with higher demand density and higher average speed. This result suggests that paratransit agencies can improve their efficiency by locating their stations closer to the areas of higher user density, providing training to the drivers and adopting new types of vehicles for faster passenger loading and unloading. The analysis, however, could not confirm the effect of the scheduling method used by a paratransit agency in generating daily service runs.
- The analysis does not show any significant difference between the ordinary linear regression method and the bootstrap method, which has been proposed in literature to address the issue that DEA is sensitive to random errors. This result could be interpreted as the robustness of the efficiency estimates from DEA and the regression model.

This research is limited in a number of aspects because of limited availability of operating data. Future research is needed and should focus on the following areas:

- It would be valuable to conduct a survey of paratransit systems to obtain more accurate and detailed information on inputs, outputs, environmental factors, service management factors, and distinctive operating practices.
- It is necessary to perform more extensive analysis of the sources of efficiency and inefficiency and influencing factors. It is important of investigate the effects of other independent variables on paratransit efficiency, such as peak-base ratio of demand or fleet, dedicated service versus nondedicated service ratio, whether or not the employees are unionized, and so forth.
- Future efforts should also be devoted to the development of guidelines that paratransit agencies can use to improve their service performance.
- Economic efficiency of a paratransit system is also of critical importance to the transit industry and the future development of paratransit services. Technical efficiency score calculated using DEA is a "relative" score, representing how well a firm is operating compared with its peers, and it is not necessarily related to the economic efficiency of a system.

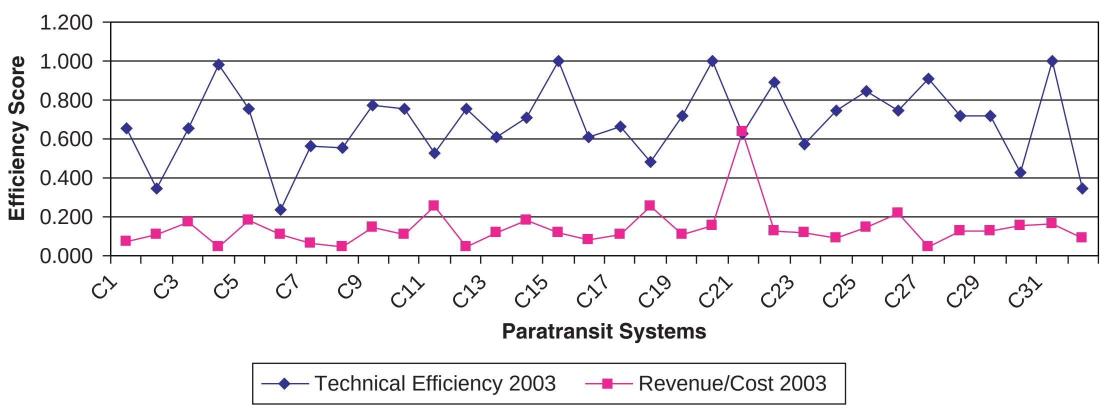

FIGURE 1 Technical efficiency and revenue-to-cost ratio.

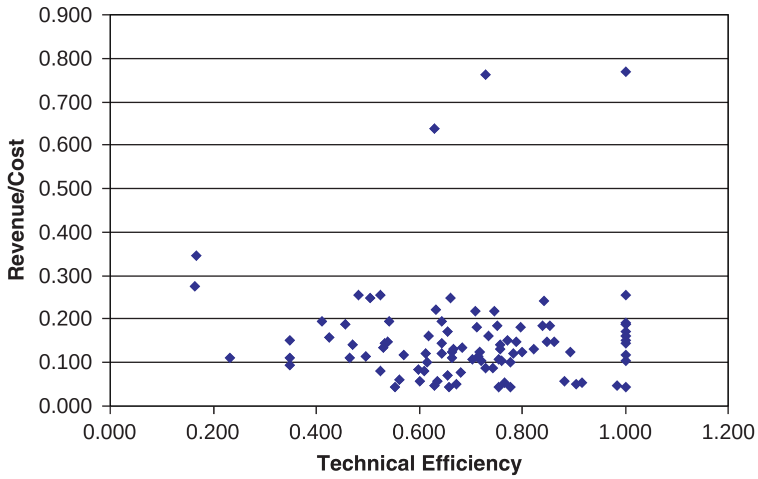

FIGURE 2 Technical efficiency versus revenue-to-cost ratio.

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REFERENCES

1. Lave, R., and R. Mathias. *State of the Art of Paratransit.* In *Transportation in the New Millennium: State of the Art and Future Directions* (CD-ROM), TRB, National Research Council, Washington, D.C., 2000.
2. Gong, B., and R. Sickles. Finite Sample Evidence on the Performance of Stochastic Frontiers and Data Envelopment Analysis Using Panel Data. *Journal of Economics,* Vol. 51, 1992, pp. 259–284.
3. Charnes, A., W. W. Cooper, and E. Rhodes. Measuring Efficiency of Decision Making Units. *European Journal of Operational Research,* Vol. 3, 1978, pp. 429–444.
4. Farrell, M. The Measurement of Productive Efficiency. *Journal of the Royal Statistical Society,* Vol. 120, 1957, pp. 253–281.
5. Chu, X., G. J. Fielding, and B. W. Lamar. Measuring Transit Performance Using Data Envelopment Analysis. *Transportation Research Part A,* Vol. 26, No. 3, 1992, pp. 223–230.
6. Karlaftis, M. G. A DEA Approach for Evaluating the Efficiency and Effectiveness of Urban Transit Systems. *European Journal of Operational Research,* Vol. 152, 2004, pp. 354–364.
7. Boame, A. K. The Technical Efficiency of Canadian Urban Transit Systems. *Transportation Research Part E: Logistics and Transportation Review,* Vol. 40, 2004, pp. 401–416.
8. Oum, T. H., W. G. Waters II, and C. Yu. A Survey of Productivity and Efficiency Measurement in Rail Transport. *Journal of Transport Economics and Policy,* Vol. 33, 1999, pp. 9–42.
9. Coelli, T., D. S. P. Rao, and G. E. Battese. *An Introduction to Efficiency and Productivity Analysis.* Kluwer Academic Publishers, Inc., Boston, Mass., 1998.
10. Xue, M., and P. T. Harker. *Overcoming the Inherent Dependency of DEA Efficiency Scores: a Bootstrap Approach.* Working Paper 99–17, Financial Institution Center, Wharton School, University of Pennsylvania, Philadelphia, 1999.

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